Mathematics: analysis and approaches
Standard Level
Paper 2

Name

Date: _____

1 hour 30 minutes

Instructions to candidates

- Write your name in the box above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Section A: answer all of Section A in the spaces provided.
- Section B: answer all of Section B on the answer sheets provided. Write your name on each answer sheet and attach them to this examination paper.
- Unless otherwise stated in the question, all numerical answers must be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: analysis and approaches formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[80 marks]**.

exam: 9 pages

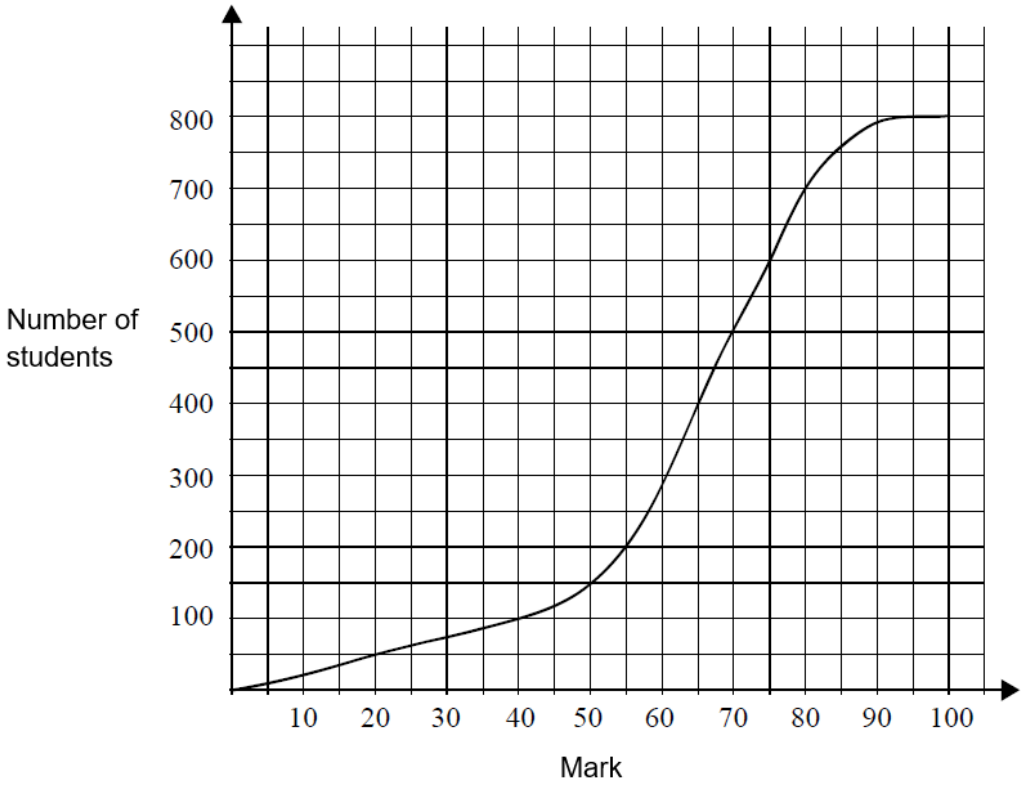
Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

Answer **all** questions in the boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 5]

A test marked out of 100 is written by 800 students. The cumulative frequency graph for the marks is shown below.



(a) Write down the number of students who scored 40 marks or less on the test. [2]

(b) The middle 50% of test results lie between marks a and b, where $a < b$. Find the value of a and the value of b. [3]

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2. [Maximum mark: 6]

A sum of \$3000 is invested at a compound interest rate of 4.6% per year.

(a) Find the value of the investment at the end of seven years. [3]

(b) The value of the investment will exceed \$5000 after x full years. Calculate the minimum value of x. [3]

3. [Maximum mark: 7]

Find the two possible lengths of AC in triangle ABC given that angle A is $42°$, $\mathrm{AB} = 12.7\,\text{cm}$ and $\mathrm{BC} = 10.2\,\text{cm}$.

4. [Maximum mark: 7]

The third term in the expansion of $(4x+p)^5$ is $160x^3$. Find the possible values of p.

5. [Maximum mark: 7]

For the students at a certain secondary school, it is determined that the time it takes to travel to school is normally distributed with mean μ and standard deviation σ. It is found that 4% of students take less than 5 minutes to get to school and 70% take less than 25 minutes. Find the value of μ and the value of σ.

6. [Maximum mark: 7]

The acceleration, in $\mathrm{m\,s^{-2}}$, of a particle at time t seconds is given by the function

$$a(t) = \frac{3}{t} + 2\sin 2t,\ t \geq 1$$

Given that the particle is at rest when $t = 1$, find the velocity of the particle when $t = 6$. [7]

Do **not** write solutions on this page.

Section B

Answer **all** the questions on the answer sheets provided. Please start each question on a new page.

7. [Maximum mark: 12]

Each day, a factory records the number (x) of boxes it produces and the total production costs (y) in dollars. The results for nine days are shown in the following table.

boxes (x)	26	44	65	43	50	31	68	46	57
costs (y)	400	582	784	625	699	448	870	537	724

(a) Let L_1 be the regression line of y on x that can be written in the form $y = mx + c$. Write down the equation for L_1. [2]

(b) Interpret the meaning of

(i) the gradient m; [1]

(ii) the y-intercept c; [1]

(c) Estimate the cost of producing 60 boxes. [3]

(d) The factory sells the boxes for \$19.99 each. Find the least number of boxes that the factory should produce in one day in order to make a profit. [3]

(e) Comment on the appropriateness of using the regression line L_1 to estimate the cost of producing 1000 boxes. [2]

8. [Maximum mark: 13]

(a) Consider the function h defined by $h(x) = \dfrac{\mathrm{e}^x}{\sqrt{x}}$, for $0 < x \leq 3$.

(i) Sketch the graph of h. [2]

(ii) Find $h'(x)$. [3]

(iii) Write down an expression representing the gradient of the normal to the curve at any point. [2]

(b) Let P be the point (x, y) on the graph of h, and Q the point $(1, 0)$.

(i) Find the gradient of (PQ) in terms of x. [2]

(ii) Given that the line (PQ) is a normal to the graph of h at the point P, find the minimum distance from point Q to the graph of h. [4]

Do **not** write solutions on this page.

9. [Maximum mark: 16]

A quiz has five questions. To pass the quiz, at least three of the questions must be answered correctly.

The probability that Sophie answers a question correctly is $\frac{1}{5}$. Let X be the number of questions that Sophie answers correctly.

(a) (i) Find $\mathrm{E}(X)$. [2]

(ii) Find the probability that Sophie passes the quiz. [4]

Isabel also takes the quiz. Let Y be the number of questions that Isabel answers correctly. The following table is the probability distribution for Y.

y	0	1	2	3	4	5
$\mathrm{P}(Y=y)$	0.67	0.05	$a+2b$	$a-b$	$2a+b$	0.04

(b) (i) Show that $4a+2b=0.24$. [2]

(ii) Given that $\mathrm{E}(Y)=1$, find the value of a and the value of b. [5]

(c) Determine which student is more likely to pass the quiz. [3]